I Organizational

- Intro
- Lectures
- Excercises → try to do them yourself!
- Exam
- Notes / books / questions
  ↳ Jackson and L.L.

II Motivation:

→ Electromagnetic waves in everyday life

→ Theory course (a glimpse into fund. theoretical physics)

III Outline of the course:

- Maxwell equations
- Methods for solving ~ Greens functions [complex calculus]
- Radiation of EM waves

- Multipole expansion [tensors]
- EM fields in Medium
- Lorentz transformations [4-vectors]
- Covariant formulation of ED
- Action principle

IV Lecture 1: Maxwell Equations

0. Vector calculus

$\vec{\nabla} f = \partial_i f \, e_i$ – gradient [in Cartesian coord.]

$\vec{\nabla} \cdot \vec{A} = \partial_i A_i$ – divergence

$(\vec{A} \cdot \vec{B}) = A_i B_i$

summation over repeated indices.

Vector $\equiv$ string of numbers $\vec{A} = (a_1, a_2, a_3)$

ε_{ijk} ~ ε-tensor 3×3×3 table of numbers

$\varepsilon_{123} = 1$, fully-anti-symmetric

Curl: $\vec{\nabla} \times \vec{A} = \varepsilon_{ijk} \nabla_j A_k e_i$

Laplacian:

$$\vec{\nabla} \cdot \vec{\nabla} \vartheta \equiv \Delta \vartheta = \partial_i \partial_i \vartheta$$

Gauss theorem

$$\int_R \vec{\nabla} \cdot \vec{E} \, dV = \oint_{\text{boundary}(R)} \vec{E} \cdot \vec{dS}$$

↳ normal vector

Stokes theorem

$$\int_S d\vec{S} \cdot (\vec{\nabla} \times \vec{B}) = \oint_{\text{boundary}(S)} d\vec{l} \cdot \vec{B}$$

Maxwell equations:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$\mu_0 = 4\pi\cdot 10^{-7}\ N\cdot A^{-2}$ magnetic permeability of the vacuum

$\varepsilon_0 \approx 8.85\cdot 10^{-12}\ \frac{A\cdot s}{V\cdot m}$ electric permittivity of the vacuum

Lorentz force

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$$

- Together these 8 + 3 differential equations determine the dynamics in CED
- We will study them for the next 3 months

- In the last lecture we will derive M.E. from some more fundamental principles, but for most of the course we postulate them and study consequences

Most basic consequences:

1. Coulomb force

$$\vec{B}=0, \quad \vec{J}=0$$

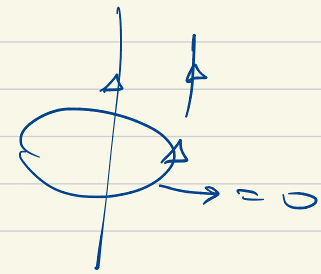
$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}\times\vec{E}=0$$

$$\vec{E}(r)=\frac{Q}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$ (spherical sym. + + Gauss theorem

$$\vec{F}=\frac{qQ}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}$$

Many particles: $$\vec{E}(r)=\sum_i\frac{q_i(\vec{r}-\vec{r}_i)}{4\pi\varepsilon_0|r-r_i|^3}$$

2. Ampere's force



$E=0$, $\rho=0$

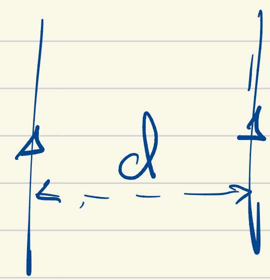
$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$\vec{\nabla} \cdot \vec{B} = 0$

take a wire along $\vec{z}$ direction:

$$\vec{B}(r) = \frac{\mu_0 I}{2\pi} \frac{\vec{e}_z \times \vec{r}}{|r|^2}$$ (using Stokes theorem)

Ampere's force:



$$\frac{\delta F}{\delta l} = -\frac{\mu_0 I I'}{2\pi d}$$

(follows from Lorentz force)

$I \simeq q \cdot V$

Note similarity
current ~ charge

3. Induction

$$\underbrace{\oint \vec{E}\cdot d\vec{l}}_{\mathcal{E}} = \underbrace{-\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}}_{\frac{\partial \varphi_s}{\partial t}} \longleftrightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

φ_s – magnetic flux through surface S

Time-varying magnetic flux generates electromotive force $\mathcal{E}$ around the boundary.

This may generate a current in the wire.

4. Charge conservation

$$\text{I} \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\text{II} \quad \vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{B} = \partial_i \varepsilon^{ijk}\partial_j B = \underbrace{\varepsilon^{ijk}}_{\text{anti-sym.}} \underbrace{\partial_i \partial_j}_{\text{sym}} B = 0$$

act with $\vec{\nabla}\cdot$ on II and ∂_t on I:

$$\partial_t \vec{\nabla}\cdot\vec{E} = \frac{\dot{\rho}}{\varepsilon_0}$$

$$\mu_0 \vec{\nabla}\cdot\vec{J} + \mu_0\varepsilon_0 \vec{\nabla}\cdot\partial_t\vec{E} = 0$$

$\vec{\partial}\cdot\vec{J} + \dot{\rho} = 0 \rightarrow$ this is the differential form of charge conservation

- If we integrate it over a volume V and use Gauss theorem we get:

$$\oint_{\partial V} \vec{J}\cdot d\vec{s} = -\dot{Q} \qquad \left(Q = \int_V d^3x\,\rho\right)$$

5. E.M. waves

Consider ME in vacuum:

$$\partial_t \left[\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0 \frac{\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\times\left[\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\right]$$

$$\vec{\nabla}\cdot\vec{E} = 0.$$